**PROBLEM 07**

**KNOWN:** Steady-state temperature distribution in a one-dimensional wall is $T(x) = Ax^2 + Bx + C$, thermal conductivity, thickness.

**FIND:** Expressions for the heat fluxes at the two wall faces (x = 0,L) and the heat generation rate in the wall per unit area.

**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Homogeneous medium.

**ANALYSIS:** The appropriate form of the heat diffusion equation for these conditions is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d^2T}{dx^2}.$$

Hence, the generation rate is

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right] = -k\frac{d}{dx}\left[2Ax + B + 0\right]$$

$$\dot{q} = -k\left[2A\right] \qquad <$$

which is constant. The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x = -k\frac{dT}{dx} = -k\left[2Ax + B\right]$$

using the expression for the temperature gradient derived above. Hence, the heat fluxes are:

*Surface x=0:*

$$q''_x(0) = -kB \qquad <$$

*Surface x=L:*

$$q''_x(L) = -k\left[2AL + B\right]. \qquad <$$

**COMMENTS:** (1) From an overall energy balance on the wall, find

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_g = 0$$
$$q''_x(0) - q''_x(L) + \dot{E}''_g = (-kB) - (-k)\left[2AL + B\right] + \dot{E}''_g = 0$$
$$\dot{E}''_g = -2AkL.$$

From integration of the volumetric heat rate, we can also find $\dot{E}''_g$ as

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L -k\left[2A\right]dx = -k\left[2AL\right]$$

which agrees with the above, as it should.