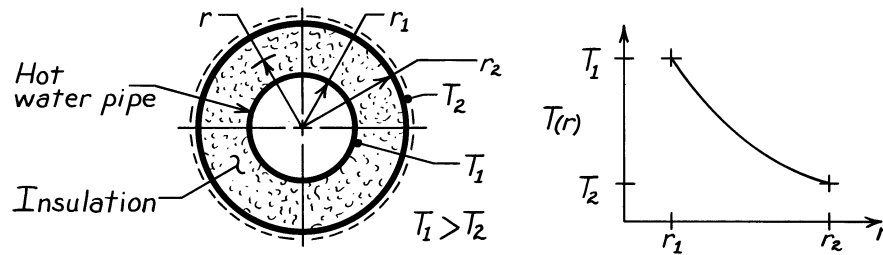


## PROBLEM 06

**KNOWN:** Hot water pipe covered with thick layer of insulation.

**FIND:** Sketch temperature distribution and give brief explanation to justify shape.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional (radial) conduction, (3) No internal heat generation, (4) Insulation has uniform properties independent of temperature and position.

**ANALYSIS:** Fourier's law, Eq. 2.1, for this one-dimensional (cylindrical) radial system has the form

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi r\ell) \frac{dT}{dr}$$

where  $A_r = 2\pi r\ell$  and  $\ell$  is the axial length of the pipe-insulation system. Recognize that for steady-state conditions with no internal heat generation, an energy balance on the system requires

$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$  since  $\dot{E}_g = \dot{E}_{\text{st}} = 0$ . Hence

$$q_r = \text{Constant.}$$

That is,  $q_r$  is independent of radius ( $r$ ). Since the thermal conductivity is also constant, it follows that

$$r \left[ \frac{dT}{dr} \right] = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient,  $dT/dr$ , and the radius,  $r$ , remains constant throughout the insulation. For our situation, the temperature distribution must appear as shown in the sketch. <

**COMMENTS:** (1) Note that, while  $q_r$  is a constant and independent of  $r$ ,  $q_r''$  is not a constant. How does  $q_r''(r)$  vary with  $r$ ? (2) Recognize that the radial temperature gradient,  $dT/dr$ , decreases with increasing radius.